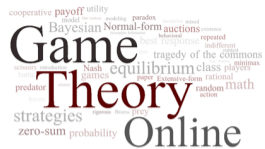




A Folk Theorem for Discounted Repeated Games

Game Theory Course:
Jackson, Leyton-Brown & Shoham



A (Simple) Folk Theorem for Discounted Repeated Games



- Consider a finite normal form game $G = (N, A, u)$.
- Let $a = (a_1, \dots, a_n)$ be a Nash equilibrium of the stage game G

If $a = (a_1, \dots, a'_n)$ is such that $u_i(a) > u_i(a')$ for all i , then there exists a discount factor $\beta < 1$, such that if $\beta_i \geq \beta$ for all i , then there exists a subgame perfect equilibrium of the infinite repetition of G that has a' played in every period on the equilibrium path.

A (Simple) Folk Theorem for Discounted Repeated Games



- Outline of the Proof:
- Play a' as long as everyone has in the past.
- If any player ever deviates, then play a forever after (Grim Trigger).
- Check that this is a subgame perfect equilibrium for high enough discount factors:

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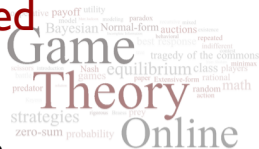
- Check that this is a subgame perfect equilibrium for high enough discount factors:
 - Playing a forever if anyone has deviated is a Nash equilibrium in any such subgame.

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 - If deviate, then given other players' strategies, the maximum possible net gain is $M - m \frac{\beta_i}{1-\beta_i}$
 - Deviation is not beneficial if $\frac{M}{m} \leq \frac{\beta_i}{1-\beta_i}$ or $\beta_i \geq \frac{M}{M+m}$ for all i .

Repeated Prisoner's Dilemma



- More complicated play: something to think about

	C	D
C	3,3	0,10
D	10,0	1,1

Repeated Games



- Players can condition future play on past actions
- Brings in many(!) equilibria: Folk Theorems
- Need key ingredients
 - Some (fast enough) observation about how others behave
 - Sufficient value to the future (limit of the means - extreme value) or high enough discount factor